ON A PERMEABLE PLATE WITH BLOWING OF GAS
INTO A COMPRESSIBLE TURBULENT BOUNDARY LAYER
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The authors have obtained the relative laws of friction and heat transfer in a compressible turbulent boundary layer with gas injection at the wall.

In [1] for supersonic flow over a permeable surface with gas blowing the following limiting formulas were obtained for the laws of friction and the critical separation parameter:

$$
\begin{gather*}
\sqrt{\Psi_{\infty}}=\int_{0}^{1} \frac{d \omega}{\sqrt{\left(1+b_{1} \omega\right)\left[\psi-\left(\psi-\psi^{*}\right) \omega-\left(\psi^{*}-1\right) \omega^{2}\right]}}  \tag{1}\\
\sqrt{b_{c r} \infty}=\int_{0}^{1} \frac{d \omega}{\sqrt{\omega\left[\psi^{\prime}-\left(\psi^{\prime}-\psi^{*}\right) \omega-\left(\psi^{*}-1\right) \omega^{2}\right]}} \tag{2}
\end{gather*}
$$

The results of numerical calculations of $E q$. (1) over a wide range of $M$ number and $\psi$ were presented in [1]. The calculation of the integral of Eq. (1) was performed as a function of the parameter $b_{1}$ for constant values of $M$ and $\psi$. References [1, 2] also suggested formulas to approximate the results of the calculations with Eqs. (1) and (2) to within an error of $15 \%$ over the range of $M=0-12$ and $\left(\psi^{*}-\psi\right)=0-30$ :

$$
\begin{gather*}
\Psi_{\infty}=\Psi_{\rho \infty} \Psi_{M \infty}\left(1-b l b_{\mathrm{cr} \infty}\right)^{2}  \tag{3}\\
b_{\mathrm{cr} \infty}=\Psi_{M \infty} \frac{1}{1-\psi}\left(\ln \frac{1+\sqrt{1-\psi}}{1-\sqrt{1-\psi}}\right)^{2} \text { for } \quad \psi<1,  \tag{4}\\
b_{\mathrm{cr} \infty}=\Psi_{M \infty} \frac{1}{\psi-1}\left(\arccos \frac{2-\psi}{\psi}\right)^{2} \text { for } \psi>1, \tag{5}
\end{gather*}
$$

where,

$$
\begin{gather*}
\sqrt{\Psi_{\rho \infty}}=\frac{2}{\sqrt{\psi / \Psi^{*}+1}} \\
\sqrt{\Psi_{M \infty}}=\frac{\operatorname{arctg} M \sqrt{r \frac{k-1}{2}}}{M \sqrt{r \frac{k-1}{2}}} ; \text { for } \quad \psi=1 b_{\mathrm{cr} \infty}=\Psi_{M_{\infty}} 4 \tag{6}
\end{gather*}
$$

In the proposed approximation (3) the quantities $\Psi_{p \infty}$ and bcro depend on the parameter $\psi$.
In the practical use of the results of the numerical calculations and the approximations obtained in [1, 2] one finds a considerable complexity, especially for the case of a given gas flow rate. The reason is that the parameter $\psi$ is not the independent variable. As follows from the heat energy balance on the surface of the permeable wall [1], the parameter $\psi$ is a single-valued function of the permeability parameter $\mathrm{b}_{\mathrm{T}}$ and of $\psi^{\prime}$ (for $\mathrm{q}_{\mathrm{rad}}=0$ ):

$$
\begin{equation*}
\psi=\frac{\psi^{*}+b_{\mathrm{r} 1} \psi^{\prime}}{1+b_{\mathrm{T} 1}} \tag{7}
\end{equation*}
$$

In addition, it can be seen from Eq. (2) that $b_{c r} \infty$ depends only on $\psi^{\prime}$, but this parameter does not appear explicitly in the proposed approximations of Eqs. (3)-(6).

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TABLE 1. Limiting Relatịve Friction Coefficients with Gas Blowing

| $\psi_{1}$ | $b_{1}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0.5 | 1 | 2 | 3,5 | 20 | 40 | 100 | 1000 |
| $M=0$ |  |  |  |  |  |  |  |  |  |
| 0,2 | 1 | 0,942 | 0,883 | 0,781 | 0,666 | 0,271 | (0,164 | 0,077 | 0,0092 |
| 0,4 | 1 | 0,904 | 0,821 | 0,695 | 0,568 | 0,206 | 0.120 | 0,055 | 0,0064 |
| 0,6 | 1 | 0,869 | 0,769 | 0,630 | 0,500 | 0,169 | 0,097 | 0,044 | 0,0050 |
| 0,8 | 1 | 0,837 | 0,725 | 0,578 | 0,450 | 0,145 | 0,083 | 0,037 | 0,0042 |
| 1 | 1 | 0,808 | 0,686 | 0,535 | 0,409 | 0,128 | 0,073 | 0,032 | 0,0037 |
| 2 | 1 | 0,693 | 0,548 | 0,399 | 0,298 | 0,083 | [0,046] | 0,020 | 0,0023 |
| $M=1$ |  |  |  |  |  |  |  |  |  |
| 0,2 | 0,896 | 0,843 | 0,789 | 0,696 | 0,592 | 0,239 | 0,144 | 0,068 | 0,0080 |
| 0,4 | 0,896 | 0,807 | 0,732 | 0.618 | 0,503 | 0,180 | 0,105 | 0,048 | 0,0055 |
| 0,6 | 0,896 | 0,775 | 0,684 | 0,558 | 0,442 | 0,148 | 0,085 | 0,038 | 0,0044 |
| 0,8 | 0,896 | 0,746 | 0,653 | 0,511 | 0,396 | 0,127 | 0,072 | 0,032 | 0,0037 |
| 1 | 0,896 | 0,719 | 0,608 | 0,473 | 0,360 | 0,111 | 0,063 | 0,028 | 0,0032 |
| 2 | 0,896 | 0,614 | 0,495 | 0,351 | 0,257 | 0,072 | 0,040 | 0,018 | 0,002 |
| $M=2$ |  |  |  |  |  |  |  |  |  |
| 0,2 | 0,69 | 0,647 | 10,604 | 0,531 | 0,449 | 0,177 | 0,106 | 0,050 | 0,0058 |
| 0,4 | 0,69 | 0,619 | 0,559 | 0,468 | 0,379 | 0,133 | 0,077 | 0,035 | 0,0040 |
| 0,6 | 0,69 | 0,593 | 0,520 | 0,422 | 0,331 | 0,109 | 0,062 | 0,028 | 0,0032 |
| 0,8 | 0,69 | 0,569 | 0,488 | 0,385 | 0,296 | 0,093 | 0,053 | 0,024 | 0,0027 |
| 1 | 0,69 | 0,548 | 0,460 | 0,355 | 0,269 | 0,082 | 0,046 | 0,020 | 0,0023 |
| 2 | 0,69 | 0,464 | 0,302 | 0,260 | 0,189 | 0,052 | 10,029 | 0,013 | 0,0014 |
| $\mathrm{M}=3$ |  |  |  |  |  |  |  |  |  |
| 0,2 | 0,509 | 0,475 | 10,442 | 0,387 | 0,325 | 0,126 | 0,075 | 0,035 | 0,0041 |
| 0,4 | 0,509 | 0,453 | 0,408 | 0,340 | 0,274 | 0,095 | 0,055 | 0,025 | 0,0028 |
| 0,6 | 0,509 | 0,433 | 0,379 | 0,305 | 0,238 | 0,077 | 0,044 | 0,020 | 0,0022 |
| 0,8 | 0,509 | 0,416 | 0,354 | 0,277 | 0,212 | 0,066 | 0,037 | 0,016 | 0,0018 |
| 1 | 0,509 | 0,399 | 0,333 | 0,255 | 0,192 | 0,057 | 0,032 | 0,014 | 0,0016 |
| 2 | 0,509 | 0,336 | 0,260 | 0,185 | 0,132 | 0,036 | 0,020\| | 0,009 | 0,0001 |
| $\mathrm{M}=4$ |  |  |  |  |  |  |  |  |  |
| 0,2 | 0,377 | 0,352 | 10,327 | 0,285 | 0,238 | 0,091 | 0,054 | 0,025 | 0,0029 |
| 0,4 | 0,377 | 0,335 | 0,300 | 0,249 | 0,200 | 0,068 | 0,039 | 0,018 | 0,0020 |
| 0,6 | 0,377 | 0,320 | 0,278 | 0,223 | 0,174 | 0,055 | 0,031 | 0,014 | 0,0016 |
| 0,8 | 0,377 | 0,306 | 0,260 | 0,202 | 0,154 | 0,047 | 0,026 | 0,012 | 0,0013 |
| 1 | 0,377 | 0,294 | 0,244 | 0,186 | 0,139 | 0,041 | 0,023 | 0,010 | 0,0011 |
| 2 | 0,377 | 0,246 | 0,1881 | 0,133 | 0,096 | 0,026 | 0,014 | 0,0062 | 0,00068 |
| $M=5$ |  |  |  |  |  |  |  |  |  |
| 0,2 | 0,287 | 0,267 | 0,2471 | 0,215 | 0.179 | 0,068 | 0,040 | 0,019 | 0,0021 |
| 0,4 | 0,287 | 0,254 | 0,227 | 0,188 | 0,150 | 0,051 | 0,029 | 0,013 | 0,0015 |
| 0,6 | 0,287 | 0,242 | 0,210 | 0,168 | 0,130 | 0,041 | 0,023 | 0,010 | 0,0011 |
| 0,8 | 0,287 | 0,231 | 0,196 | 0,152 | 0,115 | 0,035 | 0,020 | 0,0087 | 0,00096 |
| 1 | 0,287 | 0,222 | 0,184 | 0,139 | 0,104 | 0,030 | 0,017 | 0,0075 | 0,00083 |
| 2 | 0,287 | 0,185 | $\mid 0,141$ | 0,099 | 0,072 | 0,019 | 0,010\| | 0,0046 | 0,0005 |

The computation of friction and heat transfer on permeable surfaces with gas blowing can be simplified if we choose $M$ and $\psi^{\prime}$ as independent variables in $E q$. (1), and not $\psi$. For supersonic flow it is more convenient to choose, instead of $\psi^{\prime}$, the parameter $\psi_{1}$, which is connected with the stagnation enthalpy, and not with the thermodynamic enthalpy

$$
\begin{equation*}
\psi_{1}=\psi^{\prime} \frac{1}{1+\frac{k-1}{2} M^{2}} \tag{8}
\end{equation*}
$$

The results of numerical integration of Eqs. (1) and (2), taking account of Eqs. (7) and (8), are shown in Tables 1 and 2 for the case of blowing of a uniform gas with $k=1.4, r_{0}=$ 0.89 in the range $\psi_{1}=0.2-2$ and $M=0-5$. To within an error of $5 \%$ these are approximated by the formulas: for $\psi_{i}=0.2-0.9$

$$
\begin{equation*}
\Psi_{\infty}=\Psi_{M \infty}\left(1-b / b_{\operatorname{cr} \infty}\right)^{1,8} \tag{9}
\end{equation*}
$$

for $\psi_{1}=0.9-1.5$

$$
\begin{equation*}
\Psi_{\infty}=\Psi_{M \infty}\left(1-b / b_{\text {crom }}\right)^{2} \tag{10}
\end{equation*}
$$

for $\psi_{z}=1.5-2$

$$
\begin{equation*}
\Psi_{\infty}=\Psi_{M \infty}\left(1-b / b_{\mathrm{cr} \infty}\right)^{2,2} \tag{11}
\end{equation*}
$$

TABLE 2. Critical Values of the Separation Parameter with Gas Blowing

| M | $\psi_{1}$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0,2 | 0,4 | 0,6 | 0,8 | 1 | 2 | 3 | 4 | $\phi_{1}=\boldsymbol{\psi}^{*}$ |
| 0 | 10,4 | 7,1 | 5,54 | 4,63 | 4 | 2,46 | 1,71 | 1,33 | 4 |
| 1 | 8,28 | 5,71 | 4,5 | 3,75 | 3,25 | 2,01 | 1,49 | 1,19 | 3,34 |
| 2 | 6,0 | 4,16 | 3,26 | 2,72 | 2,35 | 1,44 | 1,06 | 0,84 | 2,43 |
| 3 | 4,23 | 2,91 | 2,28 | 1,89 | 1,63 | 0,99 | 0,72 | 0,58 | 1,71 |
| 4 | 3,01 | 2,07 | 1,62 | 1,34 | 1,16 | 0,7 | 0,51 | 0,4 | 1,22 |
| 5 | 2,22 | 1,52 | 1,19 | 0,99 | 0,85 | 0,51 | 0,37 | 0,29 | 0,91 |




Fig. 1. Comparison of numerical calculations of the integrals of Eqs. (1) and (2) with the approximate formulas in the range $M=0-5$ and $\psi_{1}=0.2-2$; 1) $\psi_{1}=$ 0.2 ; 2) 0.4 ; 3) 0.6 ; 4) 0.8 ; 5) 1.0 ; 6) 2; 7, 8, 9) calculations from Eqs. (9), (10), and (11), respectively; 10) calculation from Eq. (12).

The calculated data for the critical separation parameter from Eq . (2) is approximated with an error of less than $10 \%$ by the formula

$$
\begin{equation*}
b_{\mathrm{cr} \infty}=4 \psi_{1}^{-0,6} \Psi_{M}^{1 / 4} \tag{12}
\end{equation*}
$$

Figure la compares Eqs. (9)-(11) with values calculated from Eq. (1), and Fig. Ib compares Eq. (12) with Eq. (2).

In contrast with Eq. (3), Eqs. (9)-(11) do not have the unknown parameter $\psi$, while bcro is the true separation parameter and depends only on $\psi_{2}$. Therefore, the solution of the integral momentum relation on the permeable surface [1] is appreciably simplified.

For the region of finite Reynolds number it is suggested in [1] that the critical separation parameter be calculated from the integral ( $\omega_{1} \rightarrow 0$ )

$$
\begin{equation*}
\sqrt{b_{c r}}=\frac{1}{Z} \int_{0}^{1} \sqrt{\frac{1+2 \xi}{\omega\left[\psi^{\prime}-\left(\psi^{\prime}-\psi^{*}\right) \omega-\left(\psi^{*}-1\right) \omega^{2}\right]}} d \omega \tag{13}
\end{equation*}
$$

Fig. 2. Comparison of calcu-
lations of the critical blow-
ing parameters from Eqs. (15)
and (17) with experiment: 1)
calculation from Eq. (15) for
$\psi_{2}=1 ; 2,3,4,5,6$ ) calcula-
tion from Eq. (17) for $\psi_{1}=1$,
$0.66,0.34,0.24$, and 7 , re-
spectively; experiment: 7)
data of [8]; 8) data of [6];
9) data of [16].

The solution of Eq. (13) in the first approximation, when

$$
\begin{equation*}
Z=1, \omega=\left(1+1,25 \sqrt{0.5 c_{f 0}} \ln \xi\right)^{2}, \sqrt{0.5 c_{f 0}}=\left(2,5 \ln \mathrm{Re}^{* *}+3.8\right)^{-1} \tag{14}
\end{equation*}
$$

was obtained in $[1,2]$. The results of the numerical calculations were approximated by the formula [1]

$$
\begin{equation*}
b_{\mathrm{cr}}=b_{\mathrm{cr} \infty}\left(1+0.83 \mathrm{Re}^{* *-0.14}\right) \tag{15}
\end{equation*}
$$

The solution of Eq. (13) in the second approximation will be ( $Z \neq 1$ )

$$
Z=1-\omega_{10}=1-\varphi_{10} \sqrt{\Psi_{M_{\infty}} \Psi_{\rho \infty} c_{f 0} / 2}
$$

On the basis of the experimental data of [3-5] we have $\varphi_{10}=7.8$.
The results of calculating Eq. (13), allowing for Eqs. (14) and (16) in the range Re** $=$ $10^{3}-10^{5}, \psi_{1}=0.1-2$, and $M=0-5$ can be approximated to within an error of $15 \%$ by the expression

$$
\begin{equation*}
b_{\mathrm{cr}}=4 \psi_{1}^{-0.6} \Psi_{M}^{1,4}\left(1+5.3 \psi_{1}^{-0.35} \cdot 10^{-\frac{M}{11}} \mathrm{Re}^{* *-0.18}\right) \tag{17}
\end{equation*}
$$

Figure 2 compares calculations from Eq. (17) with the most reliable measurements of the critical separation parameters for supersonic flow over permeable surfaces with blowing of unfform and nonuniform gases [6-8]. The rather accurate test data of [9] are not shown in Fig. 2, since they refer to blowing of a weak aqueous solution of alkali into a concentrated aqueous solution of salt or citric acid ( $\operatorname{Pr} \approx 6, \psi_{1}=1.2-1.3$ ).

For supersonic flow over a permeable plate the authors know of no test data on critical separation parameters.

It can be seen from Fig. 2 that Eq. (17) agrees satisfactorily with experiment.
It is proposed to calculate the friction law in the region of finite Reynolds number, as was done in [1], from Eqs. (9)-(11), by replacing the limiting values of $b_{c r o m}$ and $\Psi_{M \infty}$ by their actual values, allowing for the influence of the finite Reynolds number on ber from the improved Eq. (17), and on $\Psi_{M o}$ according to [1]. In particular, for $\psi_{1}=0.9-1.5$ we obtain the expression

$$
\begin{equation*}
\Psi=\Psi_{M}\left(1-b / b_{c r}\right)^{2} \tag{18}
\end{equation*}
$$

Figure 3a compares the calculations from Eqs. (18) and (17) with the most reliable experimental measurements of relative friction coefficients in supersonic and subsonic flow over permeable surfaces with blowing of uniform and nonuniform gases. In subsonic flow of an isothermal zero-gradient stream over a permeable plate with blowing of gas at the wall there are reliable measurements of relative friction coefficients obtained in [8, 10-13]. In $[8,10]$ special steps were taken to minimize possible errors associated with nonuniformity


Fig. 3. Comparison of the calculations of relative coefficients of friction and heat transfer with experiments: 1) calculation from Eqs. (18) and (17); 2) calculation from Eqs. (19) and (21); experiment: 3) data of [2]; 4) data of [10]; 5) data of [13] for $M=3.18, \psi=\psi^{*} ; 6,7$ ) data of [15] for $M=2.5$; $3.5 ; \psi=\psi^{*}$; 8) data of [18] for $M=$ 1.8; $\psi=\psi^{*} ; 9,10$, and 11) data of [19] for $M=2 ; 3 ; 4$; $\left.\psi_{1}=0.9-1.5 ; 12\right)$ data of [16] for $M<1 ; \psi_{1}=0.9-1.1$.
of the plate permeability ( $2 \%$ in [8] and $6 \%$ in [10]), with the presence of a pressure gradient ( $\pm 0.5$ of water column), with the influence of wall roughness or increased degree of flow turbulence ( $0.2 \%$ ). Reference [8] used a direct method of measuring friction with a floating element. The error in measuring the small friction forces in [8] was reduced appreciably in comparison with the sensors previously used for this purpose in [13], by reducing the gap around the sensor. In [8] this gap was constant and equal to 0.076 mm , and in [13] it varied from 0.19 to 1.9 mm .

In the opinion of the authors of [11] the earlier measurements of relative friction coefficient on the same equipment as in [8], presented in the papers of Micley and Davis, Kendall and Rubesin, and Dahm and Mendenhall, and also the test data of McQuaid are unreliable, for a number of reasons. The author of [14] himself believes his measurements to be unreliable for blowing parameters $\mathrm{b}>1.5$.

For supersonic flow over permeable surfaces with gas blowing there are unfortunately no reliable experimental data in the literature on measurement of relative friction coefficients over a wide range of blowing parameters, as for subsonic flow over plates. Therefore, Fig. 3a for supersonic flow over a plate with wall blowing of gas presents only experimental data on friction coefficients which in the opinion of the authors of [12, 13, 15] have no very large measurement error and which could be reduced in the suggested form.

It can be seen from Fig. 3a that the proposed method for calculating the friction coeffcients agrees satisfactorily with experiment.

We now consider heat transfer under conditions of supersonic flow over a permeable plate with gas blowing. It was shown in [1] that for zero-gradient flow of a gas over a plate there is similarity of the relative laws of friction and heat transfer for $\operatorname{Pr}=\operatorname{Pr}_{T}=1$. Under these conditions Eqs. (9)-(11) and (18) could be used for the relative heat-transfer law, if we replace the parameter $b$ by $b_{T}$. In particular, for $\psi_{2}=0.9-1.5$ in the region of finite Reynolds number Eq. (18) is rewritten in the form

$$
\begin{equation*}
\Psi_{\mathrm{T}}=\Psi_{M}\left(1-b_{\mathrm{r}} / b_{\mathrm{rcr}}\right)^{2} \tag{19}
\end{equation*}
$$

Between the parameters $b$ and $b_{T}$ there is the single-valued relation:

$$
\begin{equation*}
b_{\mathrm{T}}=b \operatorname{Pr}^{n}, \tag{20}
\end{equation*}
$$

where $\mathrm{n}=0.6$ [7]. The parameter $\mathrm{b}_{\text {Tcr }}$ can be calculated from values of $\mathrm{b}_{\mathrm{cr}}$ from Eq. (17):

$$
\begin{equation*}
b_{\mathrm{rcr}}=4 \mathrm{Pr}^{0.6} \psi_{1}^{-0.6} \Psi_{M}^{1,4}\left(1+5.3 \psi_{1}^{-0.35} 10^{-\frac{M}{11}} \mathrm{Re}^{* *-0.18}\right) . \tag{21}
\end{equation*}
$$

Figure 3 b compares Eqs. (19) and (21) with the experimental data of [16-18] under conditions where Mach number varied from 0.03 to 4 and $\psi_{1}$ varied from 0.8 to 1.6.


Fig. 4. Comparison of the measured recovery factors with theory: 1, 2) calculated from Eq. (19) for $M=3$ using the test data for $\omega_{1}$ in the upper and lower transition zones, respectively, as a function of the blowing parameter; 3-6) calculated from Eq. (21) for Re** $=5 \cdot 10^{3}$ : 3) for $M=3 ; \psi_{1}=1.5$; 4) for $M=2 ; \psi=$ $\Psi^{*}$; 5) for $M=3$; $\Psi=\Psi^{*} ; 6$ ) for $M=4$; $\psi=\psi^{*}$; experiment: 7) data of [24] for $\mathrm{M}=4.35$; 8) data of [19] for $\mathrm{M}=2$, 3 , $4 ; 9)$ data of [21] for $M=2.5$; 10) data [22] for $M=3.2$; 2 .

It can be seen from Figs. 3 that the proposed method of calculating relative heat-transfer coefficients agrees satisfactorily with experiment.

In the present reduction of test data on heat transfer on permeable surfaces with gas blowing in a supersonic stream, the recovery factor was assumed equal to that on an impermeable surface.

Thus, it has been shown that in the use of the improved formulas for relative coefficients of heat transfer (Eq. (19)) and critical separation parameters (Eqs. (17) and (21)) one need not take into account the dependence of the recovery factor on the blowing parameter.

It follows from the theoretical analysis made in $[19,20]$ that gas blowing has a negligible influence on the recovery factor. According to [19], for $\operatorname{Pr}_{T}=1$ as the blowing parameter increases the recovery factor first falls to $r=0.85$, and then increases to $r=1$ (as $b_{1} \rightarrow \infty \omega_{1} \rightarrow 0$ ). An analogous result was also obtained in [20]. For the recovery factor on a permeable wall with gas blowing reference [20] obtained the expression $\left(\operatorname{Pr}_{T}=1\right)$

$$
\begin{equation*}
r=1-(1-\operatorname{Pr}) \frac{\omega_{1}^{2}\left(1+b_{1}\right)}{1+b_{1} \omega_{1}} \tag{22}
\end{equation*}
$$

For $b_{1}=0$ from Eq. (22) we derive the well-known formula for the recovery factor on an impermeable wall [20]

$$
\begin{equation*}
r_{0}=1-(1-\operatorname{Pr}) \omega_{10}^{2} \simeq \sqrt[3]{\overline{\operatorname{Pr}}} \tag{23}
\end{equation*}
$$

and for $b_{1} \rightarrow \infty$ and $\omega_{1} \rightarrow 0$ from Eq. (22) we find that $r \rightarrow 1$.
Figure 4 compares the theoretical values from [19] for $\operatorname{Pr}_{T}=1$ with the experimental values of recovery factor on permeable surfaces with gas blowing, obtained in [19-24]. Figure 4 also shows the calculated values of the critical separation parameters from Eq. (17) for $M=2,3,4, \psi=\psi^{*}$ and $R \triangleright * *=5 \cdot 10^{3}$, and also the critical value of the separation parameter for $M=3$ and $\psi_{2}=1.5$, since the experimental values of recovery factor in [19] (point 8 in Fig. 4) refer to this case.

As can be seen from Fig. 4, up to the blowing parameter values $b_{T M}=2$ there is negligible scatter of the experimental data on $r$, and the data agree satisfactorily with the theory [19]. Some of the test data of [19, 21, 22] correspond to separated boundary-layer conditions. The accuracy of measurement of recovery factor in such conditions is low, since it is known that the evaluation in the test of the values of $b$ corresponding to the condition $T^{\prime}=T_{W}$ for $\psi^{\prime}>1$ is very imprecise even for subsonic flow over a plate [16]. Therefore, one should be very cautious in regard to the test data on $r$ close to conditions of boundary layer separation. To a large extent one can determine the critical values of the blowing parameter from these data, but not the recovery factor.

Thus, the improvement made here in the relative laws of friction and heat transfer, together with the theoretical estimates of recovery factors, allow us, within the limits of the assumptions made, as a first approximation to neglect the influence of gas blowing on the recovery factor in calculating heat transfer in supersonic flow over permeable surfaces with a turbulent gas boundary layer.

## NOTATION

$\psi=i_{W} / i_{0} ; \psi^{\prime}=i^{\prime} / i_{0} ; \psi_{1}=i^{\prime} / i_{00} ; \psi^{*}=i_{W}^{*} / i_{0}$, relative enthalpies; $i_{W}$, enthalpy of the gas at the wall temperature, $\mathrm{J} / \mathrm{kg} ; \mathrm{i}^{\prime}$, enthalpy of the blown gas, $\mathrm{J} / \mathrm{kg}$; $i_{0}, i_{00}$, thermodynamic and stagnation enthalpy, respectively, of the gas at the boundary layer edge, J/kg; $i_{\mathrm{w}}^{*}$, equilibrium gas enthalpy at the wall, $\mathrm{J} / \mathrm{kg} ; \Psi=\mathrm{c}_{\mathrm{f}} / \mathrm{cf}_{\mathrm{f}}$, relative friction_law; $\Psi_{\mathrm{T}}=\mathrm{St} /$ Sto, relative law of heat transfer; M, Mach number; $k$, isentropic index; $b=\overline{2 j w} / \mathrm{cfo}_{0} ; \mathrm{b}_{2}=$
 eters; $\omega=u / u_{0}, \varphi=u / v^{*}$, relative velocities; $R e^{* *}=\rho_{o} u_{0} \delta * * / \mu_{0}$, Reynolds number; $\delta * *$, momentum loss thickness, m ; $\omega_{1}=u_{1} / \mu_{0}, \varphi_{1}=u_{1} / v^{*}$, relative velocities at the laminar sublayer edge; $v^{*}=\sqrt{\tau_{w} / \rho_{0}}$, dynamic velocity, $m / s e c ; r_{0}, r$, recovery factors on an impermeable and a permeable wall, respectively; $\xi=y / \delta$, relative transverse coordinate; subscript: cr, critical.

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## INTENSIFICATION OF CONVECTIVE HEAT EXCHANGE

IN CHANNELS WITH A POROUS HIGH-THERMAL-CONDUCTIVITY
FILLER. HEAT EXCHANGE WITH LOCAL THERMAL
EQUILIbRIUM INSIDE THE PERMEABLE MATRIX
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Results are presented from analytical and experimental studies of intensification of convective heat exchange in a channel with a porous high-thermal-conductivity filler in the case of moderate external heating.

The placement, in a channel, of a porous, high-heat-conducting material which is strongly bound to the channel walls causes a qualitative change in the mechanism and an intensification of heat transfer; heat is transferred from the channel walls by conduction through the framework inside the permeable matrix and is thence diffused in the flow as a result of intrapore heat exchange. The obvious physical concept behind this method was the reason that the development of a technology for making porous metals was accompanied by the proposal [1-5] of a large number of designs of various heat exchangers in which either the channels or the intertube space is filled with a permeable metal. Later the phenomenon of a substantial intensification of heat exchange was confirmed experimentally [6-8]. In particular, as a result of cooling provided by pumping water through a porous base, reliable operation of a laser reflector was realized at a thermal load $\mathrm{qw}=8 \cdot 10^{7} \mathrm{~W} / \mathrm{m}^{2}$ in [8]. Theoretical study of the process was held up for a long time by the absence of necessary information on the properties of permeable matrices. Recently, as data on the structural, hydraulic [9], heatexchange [10], and heat conduction [11] characteristics of different porous metals has been accumulated and generalized, there has been a rapid increase in the number of publications with analytical results [12-19]. However, not all of these works are of a qualitative, formulative nature and do not offer an exhaustive evaluation of the effect of different parameters on intensification of heat exchange in the process in question.

Formulation of the Problem. A channel of constant cross section (Fig. 1.1) of width or diameter $\delta$ is filled with a porous high-thermal-conductivity material beginning with the section $\mathrm{z}=0$. A single-phase heat carrier flows through the channel. The section $\mathrm{z}=0$ coincides with the beginning of the external heating of the walls, which is the same on both sides of the channel. The permeable matrix has perfect thermal and mechanical contact with the walls, is isotropic, and has a thermal conductivity $\lambda$ which is the same in all directions. The thermal conductivity of the heat carrier $\lambda_{0}$ is small compared to $\lambda$ (which is determined by the very essence of the method) and its thermophysical properties are constant.

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